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CRITERIA FOR REPRESENTING CIRCULAR ARC AND SINE WAVE SPAR WEBS BY NON-CURVED ELEMENTS

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INTRODUCTION

Thermal stresses are considered to be a major problem for designers and conceivers of airplanes to resolve. High speed aircraft that have been built (refs. 1 to 3) have incorporated thermal stress avoidance features in the structural component configurations. The conceivers of future aircraft also continue to try to avoid the rigid characteristics that promote large thermal stresses (refs. 4 and 5). One of the most popular thermal stress avoidance approaches is to use curved rather than flat webs for the spars. This effectively reduces the efficiency with which expansion in the spar caps is transmitted through the spar web. The result is a significantly reduced level of thermal stress.

A significant design and analysis problem ensues when the properties of these curved elements must be established for use with finite element structural models. Criteria is presented in this paper which relates the extensional and rotational characteristics of a specific dimensional circular arc and sine wave web to a flat web. This data is developed with respect to modeling details of the structure in reference 5.

SYMBOLS

A	element area
E	modulus of elasticity
F	force
f	mathematical function
I	moment of inertia
t	thickness
x, y, z	rectangular coordinates
δ	displacement
θ	rotation
Subscripts	
C	curved web
F	flat web
i, j	two discrete points
x, y, z	designates the x, y, or z direction
δ	refers to displacement
θ	refers to rotation

DESCRIPTION OF STRUCTURAL REPRESENTATION

Contrasting sketches of a flat web and a curved web spar are shown in figure 1. The spar web areas are normally at lower temperatures than the cap areas. This temperature differential between the caps and the web results in differential longitudinal expansion. The longitudinal expansion of the caps is resisted by the shear capability of the flat web which results in thermal stresses.

The level of thermal stress can be reduced by introducing a curved web. This reduction results because curved elements restrain the longitudinal thermal growth less. A very significant problem arises when this diminished restraint must be defined in simple terms for use in a finite element structural model. The primary parameters that are changed by introducing a curved web are the longitudinal (x-direction) stiffness and the resistance to rotation in the longitudinal direction (about the y-axis). The displacement in the x-direction of a straight structural element whose ends are at x=i and x=j can be expressed as:

$$\delta = \frac{F_x(x_i - x_j)}{A_x E_x} \tag{1}$$

Likewise, the rotation of a similar straight structural element with ends at x=i and x=j can be represented by the integral of the bending moment:

$$\theta = \int_j^i \frac{M_y}{E_x I_y} dx \tag{2}$$

Since the change to a curved web basically involves a reduction in axial (x-direction) displacement and in longitudinal (about the y-axis) rotation, there should exist a set of flat web to curved web reduction factors that could be used in a finite element program such as NASTRAN (ref. 6). A calculation that results in a ratio of flat to curved web displacements and rotations for pertinent geometric parameters (in this case the thickness of the web) provides a criteria for inputting equivalent stiffness into a finite element. To be more specific, if the factors

$$f_\delta = \frac{\delta_F}{\delta_C} \tag{3}$$

and

$$f_\theta = \frac{\theta_F}{\theta_C} \tag{4}$$

can be calculated, then equations (1) and (2) could be equivalently expressed as follows for the curved web case:

$$\delta_x = \frac{F_x(x_i - x_j)}{f_\delta A_x E_x} \tag{5}$$

and

$$\theta_y = \int_j^i \frac{M_y}{E_x f_\theta I_y} dx \tag{6}$$

The simplest approach to use as input to the finite element model would be to use the quantity (f_δ A_x) for the area input and the quantity (f_θ I_y) for the moment of inertia so that the curved element properties could be used for a straight element assumption.

The factors of equations (3) and (4) can be determined as shown in figure 2 by applying unit forces and unit moments to both flat and curved elements. Once the resulting displacements and rotations are known then the flat to curved ratios can be calculated.

It is important to not forget that to accomplish the simplification outlined herein, the finite element computer program has essentially been deceived. An important case in point is that since the area has been reduced to correct the axial stiffness, the computer program will consequently calculate axial stresses based on that input area. All of the axial thermal stresses calculated for the elements with reduced areas must be corrected by multiplying by the ratio of the input area to the real area. All of the axial thermal stresses in the unmodified elements (such as the caps) will be correct as calculated. Although the bending of the element is of less than prime importance, a similar problem must be considered when the moment of inertia is reduced.

DESCRIPTION OF NASTRAN MODELS

The flat web displacement and rotation due to a unit load can be calculated directly from equations (1) and (2). However, calculations for the curved web case are more rigorous. A finite element model using NASTRAN was developed for both an element of a sine wave web and a circular arc web. These models are depicted in figure 3. The models were derived for a single set of dimensions.

The distance between grid point 1 and grid point 17 is .0419 meters (1.648 inches) and the y-direction value of grid point 9 is .0121 meters (.478 inches). The radius of curvature of the circular arc is .0241 meters (.950 inches). No other dimensional parameters are presented in this paper as the data derived herein are for a specific case.

The NASTRAN models were developed with 16 elements which were all bar elements. The model contained 17 grid points, 93 degrees of freedom, 11 single point constraints, and 39 bulk data cards. The models represented the portion of the curved web from inflection point to inflection point looking into the x-y plane. Deflections and rotations resulting from unit loads were calculated using these models.

RESULTS AND DISCUSSION

The basic results obtained from the NASTRAN models are presented in figures 4 and 5. The ratio of flat to curved element displacements for a range of web thicknesses is presented in figure 4. It can be seen that the results for the sine wave web and the circular arc web are congruent for the displacement case. The ratio of flat to curved element rotations for a range of web thicknesses is similarly presented in figure 5. It can be seen that the results for the circular arc web and the sine wave web are quite close.

The data presented in figure 4 and 5 can be used as multiplying factors, as developed previously in equations (1) through (6), to represent a circular arc or sine wave spar web as a straight finite element or a series of straight elements. It is important to emphasize that the data in figures 4 and 5 are developed for the specific dimensions previously cited for the curved webs. The data are invalid for other dimensional combinations.

The data developed in this paper should be useful in obtaining axial thermal stresses and thermal deflections from a finite element model in a relatively simple mannner. It should be pointed out, however, that the assumptions do not totally describe the physical situation; therefore, calculations directed at items such as shear stresses could be erroneous.

CONCLUDING REMARKS

The basic problem of how to simply represent a curved web of a spar in a finite element structural model was addressed. The ratio of flat web to curved web axial deformations and longitudinal rotations were calculated using NASTRAN models. Multiplying factors were developed from these calculations for various web thicknesses. These multiplying factors can be applied directly to the area and moment of inertia inputs of the finite element model. This allows the thermal stress relieving configurations of sine wave and circular arc webs to be simply accounted for in finite element structural models.

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National Aeronautics and Space Administration
Edwards, Calif., October 20, 1979

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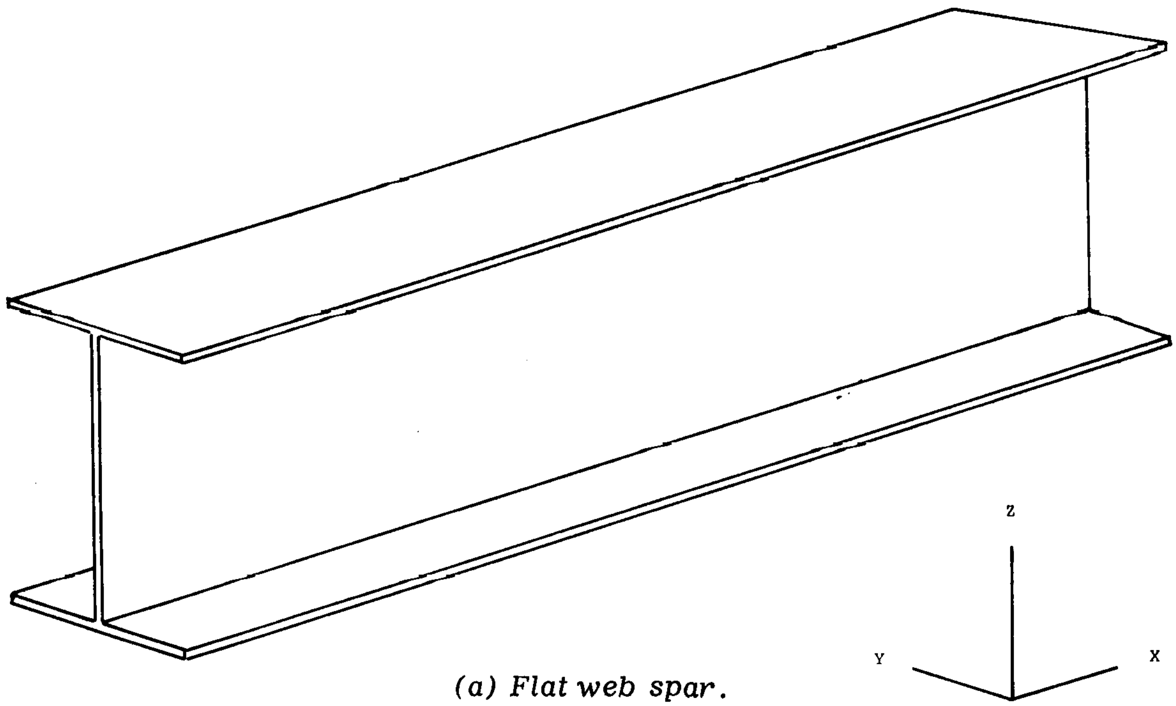
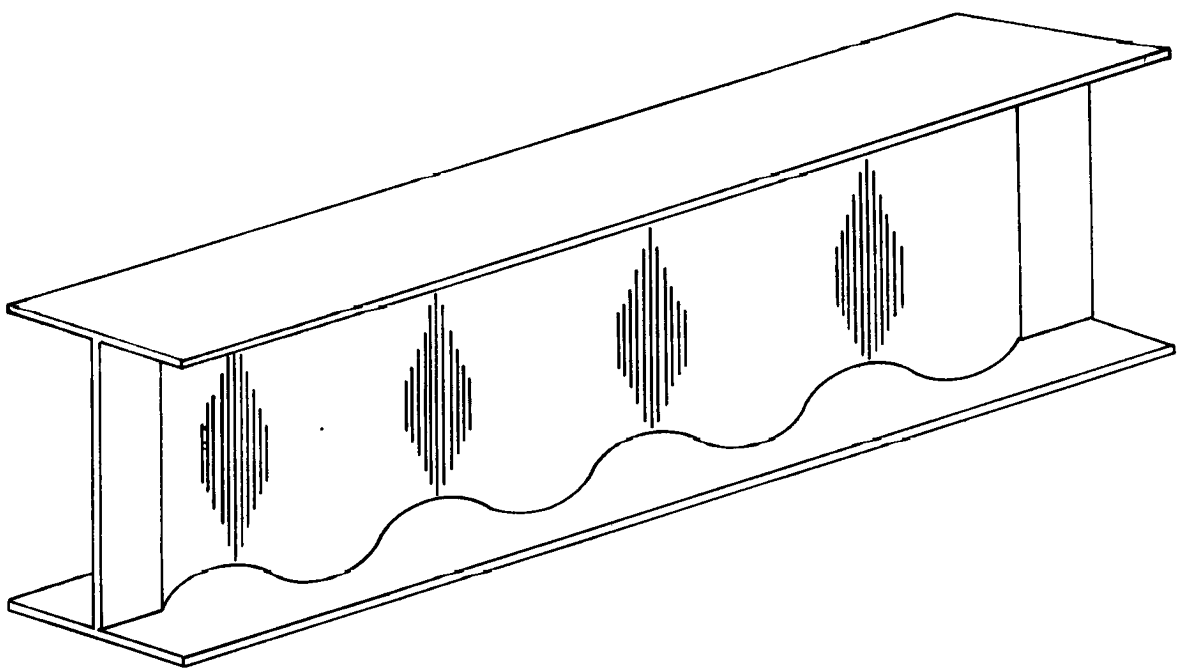
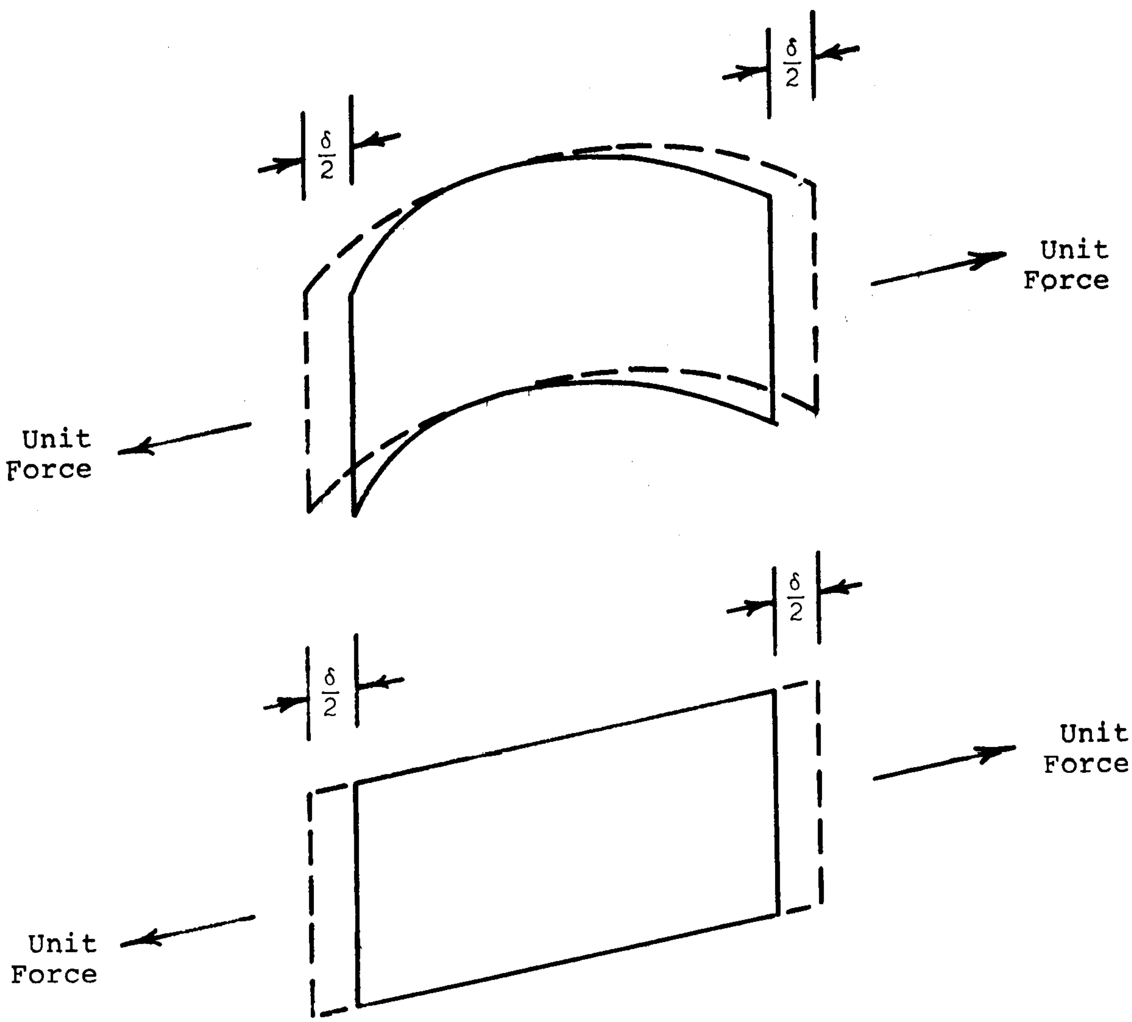


(a) Flat web spar.



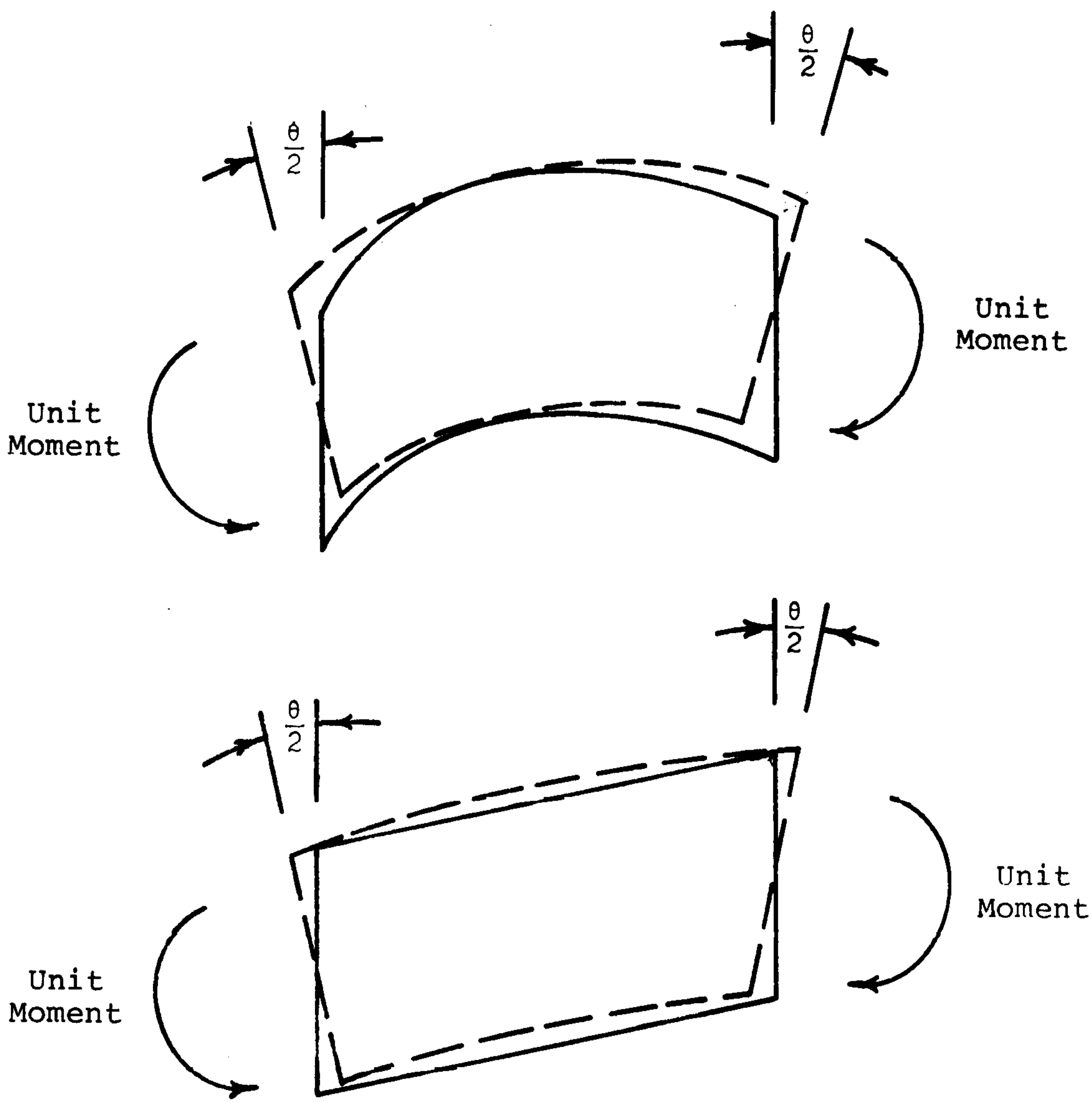
(b) Curved web spar.

Figure 1. Sketch of flat web in contrast to curved web spar.



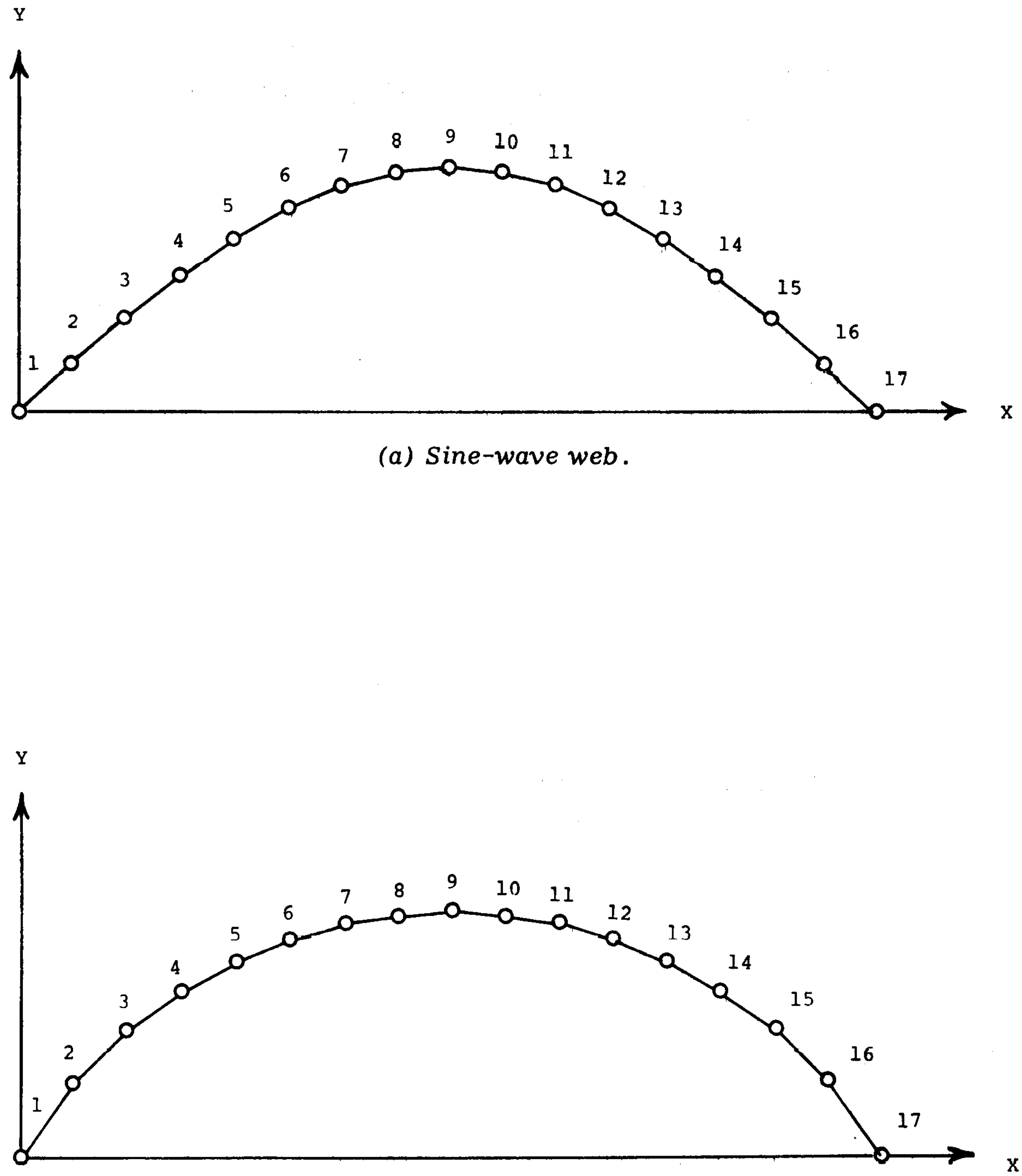
(a) Axial displacement.

Figure 2. Nomenclature of element loadings with resulting displacements and rotations.



(b) Longitudinal rotation.

Figure 2. Concluded.



(a) Sine-wave web.

(b) Circular arc web.

Figure 3. NASTRAN models representing sine-wave and circular arc configurations.

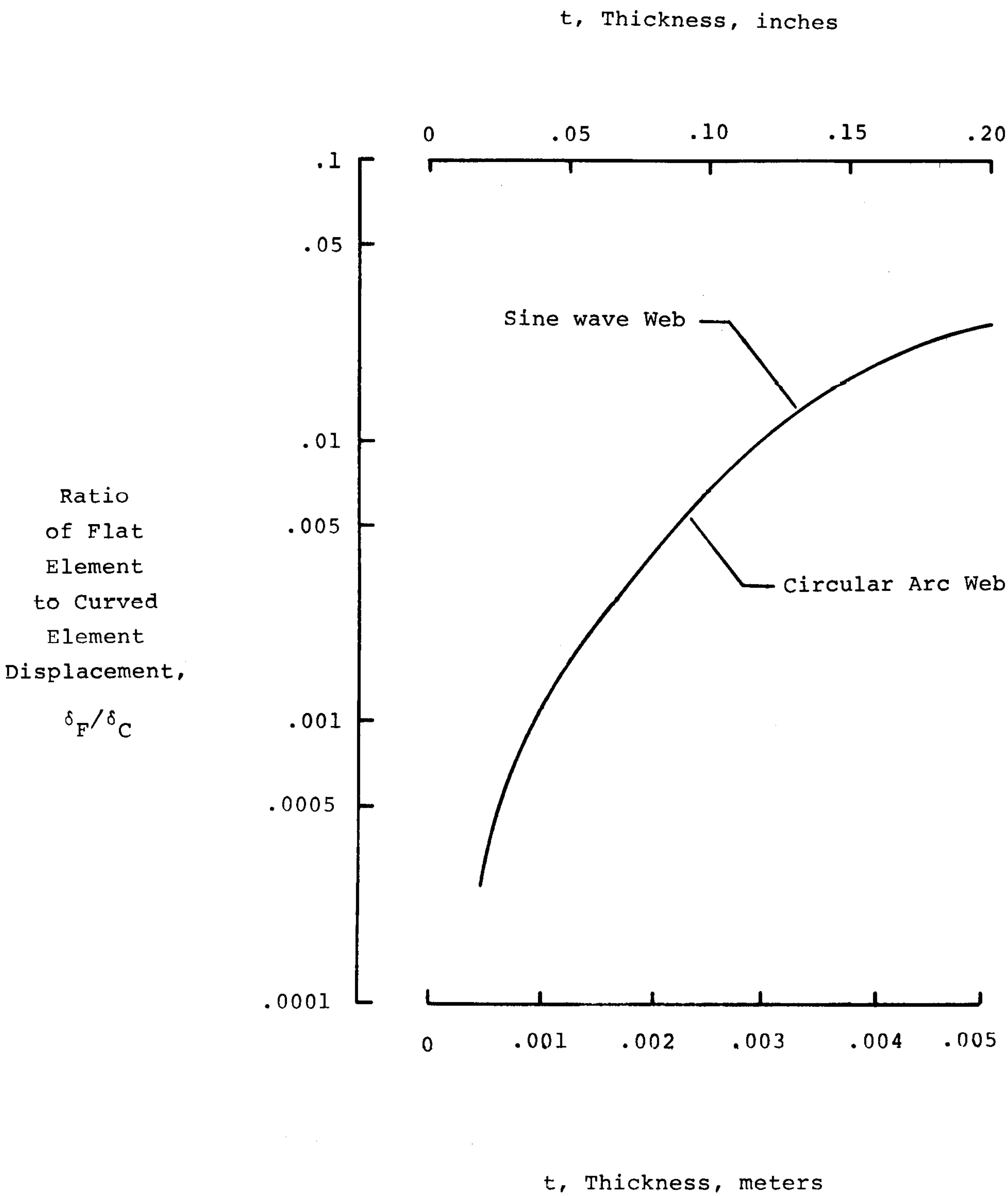


Figure 4. Effect of web thickness on the flat to curved element dispacement ratio.

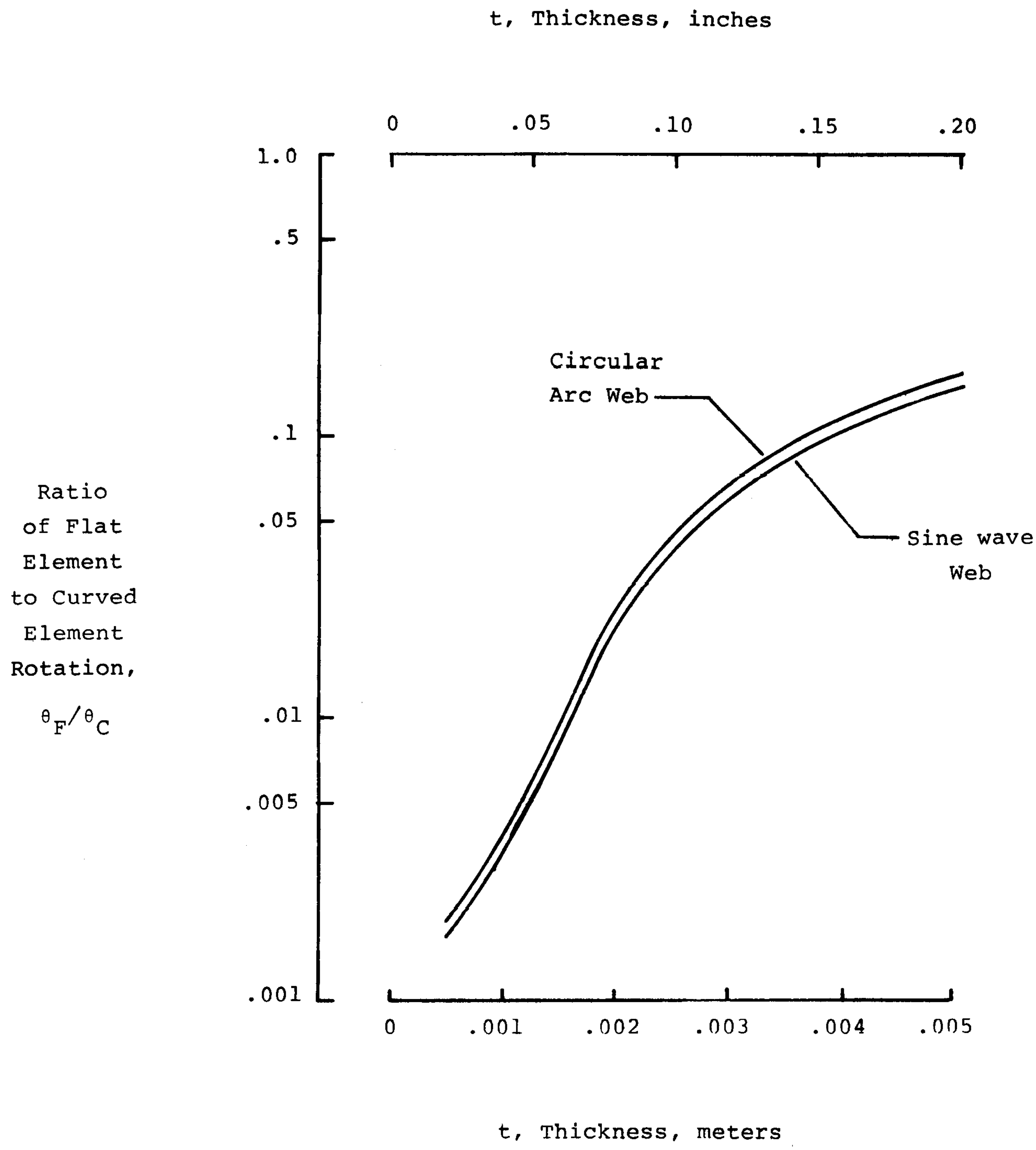


Figure 5. Effect of web thickness on the flat to curved element rotation ratio.

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16. Abstract

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